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ON THE GENERALIZED INVERSES OF LINEAR COMBINATIONS OF GENERALIZED AND HYPERGENERALIZED PROJECTORS

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Abstract
The aim of this paper is to give representations of generalized inverses of a linear combination of generalized and hypergeneralized projectors. We discuss structural properties of these projectors that make such representations possible and analyze explicit conditions for the existence of generalized inverses. Furthermore, we provide a brief overview of known results concerning generalized inverses in the context of projectors and emphasize the importance of such representations in matrix theory. Special attention is devoted to the structural properties of generalized and hypergeneralized projectors, which allow the derivation of explicit formulas for their linear combinations. Our analysis highlights connections with linear algebra, while the results may also find applications in numerical methods and related areas.
Keywords: linear combination, generalized projector, hypergeneralized projector, generalized inverses.

INTRODUCTION

Let $\mathbb{C}^{n\times m}$ denote the set of all $n\times m$ complex matrices. For a matrix $A\in\mathbb{C}^{n\times m}$, the symbols A^*, $R(A)$ and $r(A)$ will stand for the conjugate transpose matrix, range and rank of A, respectively. The symbol $\mathbb{C}_r^{n\times n}$ will stand for the set of all matrices from $\mathbb{C}^{n\times n}$ with a rank r.
By I_n and $\oplus$ we will represent the identity matrix of order n and direct sum, respectively.
The matrix $P\in\mathbb{C}^{n\times n}$ satisfying $P^2=P$ is called the projector (the idempotent matrix), until the matrix $P\in\mathbb{C}^{n\times n}$ satisfying $P^2=P=P^*$ is called the orthogonal projector. P_S denotes the orthogonal projector onto subspace S.
The Moore-Penrose inverse of A is the unique matrix $A^\dagger$ satisfying the equations:

$$(1)\ AA^\dagger A=A, (2)\ A^\dagger AA^\dagger=A^\dagger,$$
$$(3)\ (AA^\dagger)^*=AA^\dagger, (4)\ (A^\dagger A)^*=A^\dagger A.$$

The EP matrix (the range-Hermitian matrix) is the matrix $A\in\mathbb{C}^{n\times n}$ such that $A^\dagger A=A\,A^\dagger$, ie. $R(A)=R(A^*)$.
The index of a matrix $A\in\mathbb{C}^{n\times n}$, is the smallest nonnegative integer k such that $r(A^{k+1})=r(A^k)$, denoted by $Ind(A)$. For $A\in\mathbb{C}^{n\times n}$, $Ind(A)=k$, the matrix $X\in\mathbb{C}^{n\times n}$satisfying

$$(1^k)A^kXA=A^k, (2)XAX=X,$$
$$(5)\ XA=AX$$

is called the Drazin inverse of A and is denoted by $X=A^d$. If $Ind(A)=1$, then this special case of the Drazin inverse is known as the group inverse and is denoted by $A^\#$.
In 1997, Groβ and Trenkler [1] introduced generalized and hypergeneralized projectors: the generalized projector is a square matrix such that $A^2=A^*$ and the hypergenerelized projector is a square matrix such that $A^2=A^\dagger$.
We use the notations C_n^{EP}, C_n^{GP}, C_n^{HGP} for the subsets of $\mathbb{C}^{n\times n}$ consisting of EP (range-

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Hermitian) matrices, generalized, and hypergeneralized projectors, respectively.
By definition of the Moore-Penrose inverse, the group inverse and the Drazin inverse, it is easy to see that if A is the generalized or the hypergeneralized projector, then $A^{\dagger} = A^{d} = A^{\#}$. Therefore, the presented generalized inverses coincide.
In $[2-7]$, different properties of generalized and hypergeneralized projector are given. Inspired by the aforementioned results, particularly the papers [5] and [7], we present some representations of generalized inverses of linear combinations of these classes of matrices.

EXPOSITION

There are different forms of generalized and hypergeneralized projectors.
Among the useful representations of generalized and hypergeneralized projectors, the following is particularly noteworthy: any generalized projector $A \in \mathbb{C}_r^{n\times n}$ can be expressed as

$$A = Udiag(\lambda_1, \lambda_2, \dots, \lambda_n)U^*,$$

where U is a unitary matrix and $\lambda_j \in \{0,1,\omega,\varpi\}$, where $\omega = e^{\frac{2\pi i}{3}}$, are the eigenvalues of A. In the case of hypergeneralized projectors, $A \in \mathbb{C}_n^{HGP}$ if and only if

$$A = U(K \oplus 0)U^*,$$

where $U^* = U^{-1}$ and $K \in \mathbb{C}^{r\times r}$ is such that $K^3 = I_r$. From the above representations it is obvious that any generalized projector is a hypergeneralized projector.
Using the given forms of generalized and hypergeneralized projectors, one can derive representations of the generalized inverses of a linear combination of these projectors.
The form of the Moore–Penrose inverse, that is, the group inverse of a linear combination of two commuting generalized or hypergeneralized projectors, can then be determined.

Theorem 1. [5] Let $A \in \mathbb{C}^{n\times n}$ and $B \in \mathbb{C}^{n\times n}$ be commuting generalized or hypergeneralized projectors, and let $c_1, c_2 \in \mathbb{C} \setminus \{0\}$ and $c_1^3 + c_2^3 \neq 0$. Then

(2.2) $$(c_1A + c_2B)^{\dagger} = \frac{1}{c_1^3+c_2^3}(c_1^2A^2B^3 - c_1c_2AB + c_2^2A^3B^2) + \frac{1}{c_1}A^2(I_n - B^3) + \frac{1}{c_2}B^2(I_n - A^3)\ . \quad (1)$$

Furthermore, $c_1A + c_2B$ is nonsingular if and only if $n = rank(A) + rank(B) - rank(AB)$ and in this case $(c_1A + c_2B)^{-1}$ is given by (1).

As a corollary, we get that in the case when A is generalized or hypergeneralized projector and $c_1, c_2 \in \mathbb{C}, c_1 \neq 0, c_1^3 + c_2^3 \neq 0$, a linear combination $c_1I_n + c_2A$ is always nonsingular.

Theorem 2. [5] Let $A \in \mathbb{C}^{n\times n}$ be a generalized or hypergeneralized projector, $c_1, c_2 \in \mathbb{C}, c_1 \neq 0, c_1^3 + c_2^3 \neq 0$. Then $c_1I_n + c_2A$ is nonsingular and

$$(c_1I_n + c_2A)^{-1} = \frac{1}{c_1^3+c_2^3}(c_1^2A^3 - c_1c_2A + c_2^2A^2) + \frac{1}{c_1}(I_n - A^3).$$

If we consider a finite commuting family $\{A_i\}_{i=1}^m$ where all of the members are generalized or hypergeneralized projector. Hence, we have the following results.

Proposition 3. [5] Let all of $A_i \in \mathbb{C}^{n\times n}, i = \{1, \dots, m\}$ be commuting generalized or hypergeneralized projectors, $c_1, c_2 \in \mathbb{C}, c_1 \neq 0, c_1^3 + c_2^3 \neq 0$ and $k_1, \dots, k_m \in \mathbb{N}$. Then $c_1I_n + c_2\prod_{i=1}^m A_i^{k_i}$ is nonsingular.

With the additional requirements of Theorem 1 it is possible to give precise form of Moore-Penrose inverse, i.e., the group inverse.

Corollary 4. [5] Let $c_1, c_2 \in \mathbb{C} \setminus \{0\}$. If A, B are commuting generalized or hypergeneralized projectors such that $AB = 0$, then

$$(c_1A + c_2B)^{\dagger} = \frac{1}{c_1}A^2 + \frac{1}{c_2}B^2.$$

In the next result, we present the form of Moore-Penrose inverse, i.e., the group inverse of $c_1A^m + c_2A^k$, where $m, k \in \mathbb{N}$

and A is generalized or hypergeneralized projector. It is a corollary of Theorem 1.

Corollary 5. [5] Let $A \in \mathbb{C}_r^{n\times n}$ be a generalized or hypergeneralized projector and a let $c_1, c_2 \in \mathbb{C}, c_1^3 + c_2^3 \neq 0$ and $m, k \in \mathbb{N}$. Then

$$(c_1A^m + c_2A^k)^\dagger = \frac{1}{c_1^3+c_2^3}(c_1^2A^{2m} - c_1c_2A^{m+k} + c_2A^{2k}),$$

where $A^t = \begin{cases} A^3, & t \equiv_3 0, \\ A, & t \equiv_3 1. \\ A^2, & t \equiv_3 2 \end{cases}$ Furthermore, $c_1A^m + c_2A^k$ is noonsingular if and only if A is nonsingular and in this case the inverse of $c_1A^m + c_2A^k$ is given by $(c_1A^m + c_2A^k)^{-1} = \frac{1}{c_1^3+c_2^3}(c_1^2A^p - c_1c_2A^q + c_2A^r)$, where $2m \equiv_3 p, m+k \equiv_3 q$ and $2k \equiv_3 r$.

In particular, the form of the Moore–Penrose inverse, i.e., the group inverse of the linear combination $c_1A + c_2A^*$, can be specified.

Corollary 6. [5] Let $A \in \mathbb{C}_r^{n\times n}$ be a generalized projector and let $c_{1,}c_2 \in \mathbb{C}, c_1^3 + c_2^3 \neq 0$. Then
$(c_1A + c_2A^*)^\dagger = \frac{1}{c_1^3+c_2^3}(c_1^2A^2 - c_1c_2A^3 + c_2^2A)$.

Let us recall that for the matrices $A, B \in \mathbb{C}^{n\times m}$, a matrix A is less than or equal to B with respect to the star partial ordering if $A^*A = A^*B$ and $AA^* = BA^*$. If $A \in C_n^{EP}$, then for any $B \in \mathbb{C}^{n\times n}$, A is less than or equal to B with respect to the star partial ordering if and only if $AB = A^2 = BA$.
In the next theorem, we present the form of Moore-Penrose inverse, i.e., the group inverse of $c_1A^m + c_2B^k$ under the condition that A, B are generalized or hypergeneralized projectors and $AB = BA = A^2$. Remark that the same result holds if we suppose that A, B are generalized projectors such that $B - A \in C_n^{GP}$; or $A \in C_n^{EP}$, $B \in C_n^{HGP}$ such that A is less than or equal to B with respect to the star partial ordering.

Theorem 7. [5] Let $c_1, c_2 \in \mathbb{C}$, $c_2 \neq 0$, $c_1^3 + c_2^3 \neq 0$ and $m, k \in \mathbb{N}$. If $A \in \mathbb{C}^{n\times n}$ are generalized or hypergeneralized projectors such that $AB = BA = A^2$, then

$$(c_1A^m + c_2B^k)^\dagger = \frac{1}{c_1^3+c_2^3}(c_1^2A^{2m} - c_1c_2A^{m+k} + c_2^2A^{2k}) + \frac{1}{c_2}B^{2k}(I_n - A^3), \tag{2.4}$$

where $A^t = \begin{cases} A^3, & t \equiv_3 0 \\ A, & t \equiv_3 1 \\ A^2, & t \equiv_3 2 \end{cases}$ and $B^s = \begin{cases} B^3, & s \equiv_3 0 \\ B, & s \equiv_3 1 \\ B^2, & s \equiv_3 2 \end{cases}$.

In the following result, the form of the Moore–Penrose inverse, i.e., the group inverse of the linear combination $A^m(c_1A^k + c_2B^l)$, can be given.

Theorem 8. [5] Let $A \in \mathbb{C}_r^{n\times n}$ and $B \in \mathbb{C}^{n\times n}$ be commuting hypergeneralized projectors. Let $c_1, c_2 \in \mathbb{C} \setminus \{0\}, c_1^3 + c_2^3 \neq 0$ and $m, k, l \in \mathbb{N}$. Then

$$[A^m(c_1A^k + c_2B^l)]^\dagger = \frac{1}{c_1^3+c_2^3}\left(c_1^2A^{2(m+k)} - c_1c_2A^{(m+k)}B^l + c_2^2A^3B^{2l}\right) + \frac{1}{c_1}A^{2(m+k)}(I_n - B^3),$$

where $A^t = \begin{cases} A^3, & t \equiv_3 0 \\ A, & t \equiv_3 1 \\ A^2, & t \equiv_3 2 \end{cases}$ and $B^s = \begin{cases} B^3, & s \equiv_3 0 \\ B, & s \equiv_3 1 \\ B^2, & s \equiv_3 2 \end{cases}$.

Another form of generalized and hypergeneralized projectors can be stated as follows. Any generalized projector $A \in \mathbb{C}_r^{n\times n}$ can be represented by

$$A = U\begin{bmatrix} K & 0 \\ 0 & 0 \end{bmatrix}U^*, \tag{1}$$

where $U \in \mathbb{C}^{n\times n}$ is unitary and $K \in \mathbb{C}^{r\times r}$ is such that $K^3 = I_r$ and $K^* = K^{-1}$. Any hypergeneralized projector $A \in \mathbb{C}_r^{n\times n}$ has a form

$$A = U\begin{bmatrix} \sum K & 0 \\ 0 & 0 \end{bmatrix}U^*,$$

where $U \epsilon \mathbb{C}^{n\times n}$ is unitary, $\sum = diag(\sigma_1 I_{r_1}, \dots, \sigma_t I_{r_t})$ is a diagonal matrix of singular values of $A, \sigma_1 > \sigma_2 > \cdots > \sigma_t > 0, r_1 + r_2 + \cdots + r_t = r$ and $K \in \mathbb{C}^{r\times r}$ satisfies $(\sum K)^3 = I_r$ and $KK^* = I_r$.

Using these forms, further representations of the Moore–Penrose inverse, i.e., the group inverse of a linear combination of generalized and hypergeneralized projectors, can be expressed.

First, a representation of the Moore–Penrose inverse, i.e., the group inverse is given in the case of commuting generalized projectors or hypergeneralized projectors.

Theorem 9 [7] Let $A \in \mathbb{C}_r^{n\times n}$ and $B \in \mathbb{C}^{n\times n}$ be commuting generalized projectors or commuting hypergeneralized projectors, and let $m, k \in \mathbb{N}$, $c_1, c_2 \in \mathbb{C} \setminus \{0\}$ and $c_1^3 + c_2^3 \neq 0$. Then

$(c_1 A^m + c_2 B^k)^\dagger = (c_1 A^m + c_2 A A^\dagger B^k)^\dagger + c_2^{-1}(I_n - AA^\dagger)(B^k)^2$.

When the product of generalized or hypergeneralized projectors equals zero, the following representation of their linear combination is derived.

Theorem 10. [7] Let $m, k \in \mathbb{N}$, $c_1, c_2 \in \mathbb{C} \setminus \{0\}$. If $A, B \in C_n^{GP}$ or $A, B \in C_n^{HGP}$ such that $AB = 0 = BA$, then

$(c_1 A^m + c_2 B^k)^\dagger = c_1^{-1}(A^m)^2 + c_2^{-1}(B^k)^2$.

Another form of the Moore–Penrose inverse, i.e., the group inverse for the linear combination $c_1 A^m + c_2 A^k$ of generalized or hypergeneralized projectors is presented in the following proposition.

Theorem 11. [7] Let $A \in \mathbb{C}_r^{n\times n}$ be a generalized or hypergeneralized projector and let $c_1, c_2 \in \mathbb{C}$, $c_1^3 + c_2^3 \neq 0$ and $m, k \in \mathbb{N}$. Then

$(c_1 A^m + c_2 A^k)^\dagger = \frac{1}{c_1^3 + c_2^3}[c_1^2 (A^m)^2 + c_2^2 (A^k)^2 - c_1 c_2 A^m A^k]$.

The next theorem provides another form of Moore-Penrose inverse, i.e., the group inverse of $c_1 A^m + c_2 B^k$ under the condition that A, B are generalized or hypergeneralized projectors such that $AB = BA = A^2$

Theorem 12. [7] Let $c_1, c_2 \in \mathbb{C}, c_2 \neq 0, c_1^3 + c_2^3 \neq 0$ and $m, k \in \mathbb{N}$. If $A \in \mathbb{C}_r^{n\times n}$ and $B \in \mathbb{C}^{n\times n}$ be generalized projectors such that $B - A \in C_n^{GP}$ or $A \in C_n^{EP}$, $B \in C_n^{HGP}$ such that A is less than or equal to B, then

$(c_1 A^m + c_2 B^k)^\dagger = \frac{1}{c_1^3 + c_2^3}[c_1^2 (A^m)^2 + c_2^2 (A^k)^2 - c_1 c_2 A^m A^k] + c_2^{-1}(I - AA^\dagger)(B^k)^2$.

In the following result, we derive the form of the Moore–Penrose inverse, i.e., the group inverse, of the linear combination $A^m(c_1 A^k + c_2 B^l)$.

Theorem 13. Let $A \in \mathbb{C}_r^{n\times n}$ and $B \in \mathbb{C}^{n\times n}$ be commuting hypergeneralized projectors. Let $c_1, c_2 \in \mathbb{C} \setminus \{0\}, c_1^3 + c_2^3 \neq 0$ and $m, k, l \in \mathbb{N}$. Then

$$[A^m(c_1 A^k + c_2 B^l)]^\dagger = (A^m)^\dagger (c_1 A^k + c_2 A A^\dagger B^l)^\dagger + c_2^{-1}(I_n - AA^\dagger)(B^l)^\dagger. \quad (2)$$

Proof. We suppose that $A \in \mathbb{C}_r^{n\times n}$ and B are generalized projectors (the following reasoning works if A, B are hypergeneralized projectors) and that A has the form (1). Since $AB = BA$, we get that B has the following form

$$A = U\begin{bmatrix} D & 0 \\ 0 & G \end{bmatrix} U^*,$$

where $U \in \mathbb{C}^{n\times n}$ is unitary, $K \in \mathbb{C}^{r\times r}$ is the generalized projector such that $KD = DK$, and $G \in \mathbb{C}^{(n-r)\times(n-r)}$ is the generalized projector. Then

$$A^m(c_1 A^k + c_2 B^l) = U\begin{bmatrix} K^m(c_1 K^k + c_2 D^l) & 0 \\ 0 & c_2 G^l \end{bmatrix} U^*,$$

where

$$K^m = \begin{cases} I_r, m \equiv_3 0 \\ K, m \equiv_3 1, \\ K^2, m \equiv_3 2 \end{cases}$$

$$D^l = \begin{cases} P_{R(D)}, l \equiv_3 0 \\ D, \ l \equiv_3 1, \\ D^2, \ l \equiv_3 2 \end{cases}$$

and

$$G^l = \begin{cases} P_{R(G)}, l \equiv_3 0 \\ G, \ \ l \equiv_3 1. \\ G^2, \ l \equiv_3 2 \end{cases}$$

Since D^3 is an orthogonal projector and $(c_1K^k)^3 + (c_2D^l)^3 = c_1^3 I_r + c_2^3 D^3$, we get that $(c_1K^k)^3 + (c_2D^l)^3$ is nonsingular for all constants $c_1, c_2 \in \mathbb{C} \setminus \{0\}$ such that $c_1^3 + c_2^3 \neq 0$. From the invertibility of $(c_1K^k)^3 + (c_2D^l)^3$, it follows that $c_1K^k + c_2D^l$ is nonsingular. Now,

$$A^m(c_1A^k + c_2B^l)^\dagger = U \begin{bmatrix} K^{-m}(c_1K^k + c_2D^l)^{-1} & 0 \\ 0 & c_2^{-1}(G^l)^\dagger \end{bmatrix} U^*, \quad (3)$$

where

$$(G^l)^\dagger = \begin{cases} P_{R(G)}, l \equiv_3 0 \\ G^*, \ l \equiv_3 1. \\ G, \ \ l \equiv_3 2 \end{cases}$$

Since

$$AA^\dagger = U \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} U^*,$$

and

$$I_n - AA^\dagger = U \begin{bmatrix} 0 & 0 \\ 0 & I_{n-r} \end{bmatrix} U^*,$$

then the form (3) is equivalent to the form (2). This completes the proof.

Example 1. Let $A, B \in \mathbb{C}^{3\times 3}$ be defined by

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$c_1 = 2, c_2 = 1, m = l = k = 1.$

By direct calculation, we obtain $A^2 = A^\dagger = A$, $B^2 = B^\dagger = B$, hence, $A, B \in C_n^{HGP}$. Also, $AB = 0 = BA$. Now,

$$A(A + B) = 2A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

First, we will apply Theorem 8.

Since, $c_2^{-1} = 1$, $AA^\dagger = A$, $(2A)^\dagger = \frac{1}{2}$,

$$I_3 - AA^\dagger = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

we obtain

$$A(A + AA^\dagger B)^\dagger = \frac{1}{2}A = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$c_2^{-1}(I_3 - AA^\dagger)B^\dagger = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = B,$$

$$[A(2A + B)]^\dagger = \frac{1}{2}A + B = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

On the other hand, by Theorem 13, we have

$$2A + AA^\dagger B = 2A + AB = 2A + 0 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$(2A)^\dagger = \frac{1}{2}A,$$

$$c_2^{-1}(I_3 - AA^\dagger)B^\dagger = (I_3 - A)B = B,$$

$$[A(2A + B)]^\dagger = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The Moore–Penrose inverse according to Theorem 13 for this example is identical to the result obtained from Theorem 8.

CONCLUSION

In this paper, representations of generalized inverses for the linear combinations of generalized and hypergeneralized projectors have been studied. Structural properties of these projectors that allow such representations were analyzed, and explicit forms for the Moore–Penrose inverse (group inverse) were derived for certain combinations of commuting projectors. The results highlight connections with linear algebra and provide formulas that may be useful in numerical computations and related applications. Future work could explore extensions to non-commuting projectors, weighted inverses, or other generalized inverse types, as well as potential applications in computational methods and matrix theory.

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